THE REAL IN SCIENCE

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WHEN one looks over the world of natural phenomena and begins to study it in all its complexity, his usual motive is that of organizing it in some way, so that he may lose the feeling of bewilderment and the sense of being overwhelmed by the multitudinous sea, and the scornful mountains, may in some way feel that he is master of the serene clouds and the flash of lightning. He feels within himself that he is superior to these exterior things in many ways, and that if he can understand them and their ways of behaving he can control them. He is also dominated by a feeling for beauty, and the disorder he finds in his impressions of the world shocks this esthetic sense, so that he undertakes to examine the world closely to see if perchance there be not some hidden beauty in it after all. In the pursuit of these aims he describes and catalogues facts, orders them by relations that his mind finds, deduces statements that comprise many facts in small compass, generalizes these into laws, and the laws into systems of science. He endeavors to reduce the number of variables in terms of which he desires to express himself to the fewest he can. He thus comes to feel in the end that when he has succeeded in stating an unlimited number of facts in a few laws containing a few terms, and particularly when by these he can make predictions that are closely fulfilled, he has discovered the reality of nature. He thinks he has analyzed phenomena into their constituent elements, and that these elements are permanent and unchanging in their real nature, and he considers that therefore they may be correctly called realities. We may, therefore, as philosophers ask the scientist to justify his claim that he has found reality. Many scientists of late have considered the challenge. Instances are Poincaré and Enriques, who have treated the problem in masterly fashion. It is indeed the scientist himself and not the philosopher who can properly consider the problem, for science must be allowed to speak from its own standpoint, and value its results by its own standard. We may, however, all of us, pass judgment on the content science puts into the word real, and upon the precision of its statements as to what it desires to prove regarding the real.

We need too to take into account the very closely allied field of mathematics, for we must not confuse mathematics with science. They differ radically in content, method, foundations, objects, and validity

in experience. The more fundamental would seem to be mathematics, for we can scarcely eliminate it from any kind of thinking. In science particularly the dependence is great. We find, for instance, that the planets describe twisted curves through the centuries; lines of force stream like gossamer threads from electron to electron; the quivering ribbons of the aurora illuminate the pages of electrodynamics; particles of air leave streamlines with intricate turns and interwoven loops, with vortices whirling in the turbulent current; the molecule zigzags its dizzy path through the colloidal solution; wave-fronts that bend and warp fly through space with every snap of the wireless; quanta of energy, electrons, magnetons, as well as gravitational particles, if there be such, make space granular with singular points; the facets of every crystal flash with groups, electric and magnetic fields move like expanding clouds, with congruences of lines darting out like lightning flashes; the trihedral of polarized light spins like a firework; the spectrum hues are close to the roots of an infinite equation; the moving system finds a minimum variation; the mended rubber tube is full of integro-differential equations; the laws of statistics blush in the petals of every daisy; and nature in her wildest vagaries preserves the decorum of the laws of probabilities. One may study mathematics with little study of science, but one can not go far in science without the constant study of mathematics. Mathematics conditions the scientist with inflexible rules, which he must not transgress, and, at the same time, it gives him freedom from postulates that he himself at first considers binding, together with methods that are independent of the visualizing power or other concrete modes of thinking. It thus becomes evident that the answer to many an inquiry of the philosopher in the field of science may be discovered by asking the same question in the field of mathematics. The wistful soul of man has often turned to both, indeed, asking for real gems and a draught of real inspiration, after it has been deceived by the magician's fool's gold and glittering mica which it took for real wealth, or has panted long and far after the philosopher's mirage with cool waves and shady palms that only tantalized its thirst.

The real in knowledge—which does not vanish as the dew in the morning sun, or the mist in a frosty night; the real of knowledge which does not beckon like the will-o'-the-wisp to swamps and mires; the real of knowledge which does not in the eating turn to ashes and bitterness —the real of knowledge, which is as fixed as the constellations that stud the nightly sky; the real of knowledge, which is as solid as the ragged peak, defying storm, wind, lightning, and seasons; the real of knowledge which illumines the path with a flood of white light—where is it to be found? Is it only a vain hope? We turn to mathematics and can answer confidently, no. For through the turmoil and vicissitudes of

eight millenniums, through the whole enormous stretch of human history—we find arithmetic, the guide of the astrologer predicting the fortunes of the royal infant, and the companion counting the pennies of the flower girl in the streets of Babylon; noting the tale of conquered land under Rome, and the vanishing ranks of the crusaders; building the towers of modern Babel, and numbering the loaves of bread the poor shall eat to-morrow. Here is a kind of reality, at least. Again for three millenniums, we see geometry settling the property-rights of the Nilean owner, determining the blocks of the pyramids, constructing the Eiffel tower and the Quebec bridge. Here-in is surely the real. For how can that which is not real persist for thousands of years? During the last century and a half, mathematics has determined the swing of the stars in their courses, it has built telephones and telegraphs, saved the victims with wireless, and is present in every sphere of human life of to-day. There must be reality in its world, for how else could these facts be?

It is a hoary question of the race. Perhaps not arising in ancient Asia, for there the search was not for reality, but for a mystical Essence older than life and nature; but in Europe the problem was discussed many centuries ago.

There was a brilliant genius, who watched the stately march of the stars from the mountain tops of Samos, or drew diagrams on the wave-washed sands of the Icarian beach, or laid out magic squares, or charmed his select group of initiates with his lyre and the beautiful relations he had found to exist between tones and numbers. In the cube he found a symmetry of form, and in its 6 faces, 8 vertices, and 12 edges, numbers that gave the fundamental tone, the quarter, and the octave, harmony in form, in number and in music combined. How far he penetrated similar symmetries in the world about him it is hard to say, but certain it is that he saw the universal presence of mathematics, and the intuition flashed through his mind into the world that mathematics could account for the whole universe. Mountains and sea, the soul of man, and the justice in society, could be explained in this way. Triangles, squares, pentagons, spheres, cubes and other figures were certainly incarnations of number, and so were tones of the lyre, why not all the phenomena of nature? The heavens must rotate in perfect circles, and there must be a music of the spheres so subtle in its numerical structure that only the most profound could hear it. The earth must be a sphere, for was not the sphere perfect in all its parts? If he had sensed the modern group as it is present in crystals dug from the mountain cavern, he would have been even more firmly convinced that number is at the heart of every structure. But he did have the vision of mathematical concept matching natural object, mathematical

structure matching the intricacy of natural phenomena, yet all with
certainty, elegance, proportion and harmony. He saw mathematics
building a tremendous symphony out of the orchestral music of nature.
Across twenty-five centuries we see Pythagoras's majestic figure and
appreciate his far-sweeping vision.

Half a century later Heracleitos mused by his fire on the bank of an
Ephesian brook. The fuel disappeared as the flames danced and the
smoke drifted away. He watched the water hurry over the rocks in a
tumble of white foam, sweep past and vanish down the slope. Mists
rose from the Ægean Sea and became drifting clouds; they broke and
descended in rain which disappeared into the ground. He too had the
penetrating vision of the seer as he watched the glow of the coals, and
with much irony on the folly of the half-informed, the reveler, and
the fickle-minded, he drew his vision in a few immortal strokes:

> Fire lives by the death of earth, and air by the death of fire; water lives by
> the death of air, and earth by the death of water. This order, which is the same
> in all things, no one of the gods or man has made, but it was ever, is now, and
> ever shall be an everlasting fire, fixed measures of it kindling and fixed measures
> of it going out.
>
> You can not step twice into the same stream, for ever fresh waters flow in
> upon you.
>
> The quick and the dead, the waking and the sleeping, the young and the old,
> are the same; for the former are changed into the latter and the latter into the
> former.

Is this only vague Hellenic metaphysics? Salt disappears in water
and we have neither salt nor water; the molecule breaks up into atoms,
the atom into electrons and nucleus, which are all in incessant motion,
and occasionally explode so that new atoms are formed. The nucleus
shrinks and expands and may itself be a complex system. Energy
flows out into space and is dissipated, whither who knows? Nor whether
it may not be regathered into blazing suns! The Brownian flashes, the
pulsating cell, the evanescent process of mind, all prove that the uni-
verse is incessant change, and that it is extremely unlikely that any
one of its configurations is ever repeated. The song vanishes with the
singing, the landscape with the seeing, the dream with the dreaming.

> Onward and on, the eternal Pan,
> Who layeth the world's incessant plan,
> Halteth never in one shape,
> But forever doth escape,
> Like wave or flame, into new forms,
> Of gem, and air, of plants and worms.

Did not Heracleitos have a vision of the evolving universe, wherein
all is in motion and motion is in all, whether particle of ether or propa-
gation of an insubstantial state, or the intermingling fields of electric
and magnetic action, or the creative evolution of electron, matter, and
life?

Yet another half century and we stand with Parmenides on the shores of the blue Mediterranean, the same day after day and night after night despite dancing waves and casual sail; admiring the golden sunset that came every evening, the same sun in spite of storm and wind; the stars in rigid constellations traversed the dark blue sky, preserving their paths despite the few wanderers; the Italian mountains in the distance always the same despite the shifting lavender, amethyst and purple on their slopes. Austere and dignified one must be who sees things like these, and what he thought he put into a poem:

The world is, and it is impossible for anything not to be. For one can not know what is not—that is impossible—nor utter it, for it is the same thing that can be thought and that can be. It needs must be that what can be thought and spoken of is; for it is possible for it to be, and it is not possible for what is nothing to be. What is, is uncreated and indestructible, alone, complete, immovable, and without end. Nor was it ever, nor *will it be;* for now *it is,* all at once, a continuous one. How can what *is* be going to be in the future? or how could it come into being? If it come into being it is not; nor is it going to be in the future. Thus is becoming extinguished and passing away not to be heard of. Moreover it is immovable in the bonds of mighty chains, without beginning and without end; since coming into being and passing away have been driven afar, and true belief has cast them away. And there is not and never shall be any time other than that which is present. Wherefore all these things are but the names which mortals have given, believing them to be true—coming into being and passing away, being and not being, change of place and alteration of bright color.

But this too is modern science, for is not energy always energy, helium always helium, electrons always electrons, momentum always momentum, ether always ether? From acorns we always have oaks, and from thorns never grapes nor figs from thistles. The wave runs over the field of bending grain, but the motion is illusion, for the grain is always there; glacier and sea and cloud seem diverse, but are in reality only forms of the same water; the moving picture is a clever trick, which shows that all motion may be mere illusion; actual diversity of substance no longer exists in chemistry, the only diversity being one of combination, just as groups of men collect, and separate to form new groups. We can very well agree with the vision of Parmenides as he looked across the sea from his lofty summit, for the aim of modern science is to discover if it can the ultimate realities in terms of which all the forms of nature may be stated. For this reason it invented an ether, in which indestructible, uncreatable vortices constituted matter, stresses constituted electricity, while all the phenomena of electromagnetism were due merely to ether flow. It is for this reason that science seeks to reduce all the phenomena of life to the nicely balanced play of forces that are already in the system and to account for behavior in the same terms. This is the monistic dynamical view of the universe. We need not pause to trace its forms throughout the twenty-

four centuries since Parmenides, nor the forms of the views of Heracleitos or Pythagoras, for they have appeared many times as new philosophies, more acutely stated but not more far-reaching.

Daring thinkers they were, who had no degrees, no elaborate equipment, no research programs, no reference libraries! They faced the sphinx and from her mysterious lips heard answers which were contradictory and yet all correct. Who denies to-day that the world has a mathematical structure, from the electrons in their orbits and the quanta flung off from the radiating vacuum? Who denies the universal validity of the laws of conservation of energy, of momentum, or varying action? Who does not see the instantaneous evanescence of all the phenomena of experience as the mighty pendulum of eternity ticks off the seconds of the universe? Whence? Whither? What? Is the world but the projection of a mighty lantern on the screen of the senses? Are we tricked into imagining there is something real and permanent in the universe? Is the mathematical structure after all the only permanent part of the universe, and the other seeming realities merely aboriginal and naïve delusions, like the ghosts, goblins, elves and phantasms of the past? When we read modern works on electro-dynamics, we seem indeed very close to this view, for the geometry of a non-Euclidean four-dimensional space seems amply able to state every known phenomenon of physics and chemistry. Or does indeed the mathematician dream, and in his dream see a fairyland of frost which is too beautiful and too fragile to exist in the sunlight of prosaic every-day life? Many things once thought real have vanished. The ether was so real that its density was about that of air on the summit of a mountain two hundred miles high, its rigidity about a billion times that of steel, yet to-day what is the ether? The mass of a body was once supposed to persist through the most fiery tests, yet to-day it varies with every change in velocity, and may be merely a number which is zero when there is no velocity. Space at least was supposed to be exempt from the vicissitudes of public opinion, and the intense cold and emptiness of interstellar space has chilled many an emotional mind. Yet to-day we have our choice of three incompatible spaces for the universe to exist in, with absolutely no way of ever deciding in which we really live. Time was supposed to be almost the last foundation of the world, even under the series of caryatides that supported the universe, but time is for science to-day a local phenomenon, so that we do not know what the same time in Europe and America can mean, and events may happen which are neither simultaneous nor yet one before the other. The dimensions of space seem a fundamental reality, yet we do not know whether we live in four dimensions or more, or simply three. Inspect the list of terms from modern science closely: ether, electron,

energy, mass, space, time, dimensionality, and we might add many more. Do they represent realities or are they merely fancies of our too easily illusioned minds? Where is the criterion we can apply with some assurance of certainty?

Is it in the senses through which all observations are made? Who then has seen the ether, or space, or a wireless wave? Who has heard energy or put entropy into a vial to be smelled or tasted? What sense feels the X-ray, or what finger can wind up the magnetic line of force? Who ever moved a Faraday tube with its ends fixed—that reality of which Thomson builds all electrodynamics? Who can detect gravity by his senses as it swings the stars along their ponderous curves? Even if recording instruments of every type, cameras, chronographs, or automatic apparatus of every description, had for thousands of years kept as faithful an account as the recording angel, nowhere should we find in these records energy, space, waves, entropy, temperature, fields of force, life or mind. None of these is to be grasped by the senses. Even number—that is not given by the senses either. I sit by the table and watch the flashing scales of the goldfish in its bowl. I see one fish through the side of the bowl, another through the top of the water. My finger reports one fish, my eye two. Feature to feature, shining scale to scale, motion to motion, the two fish are exactly alike. Does my eye report reality if there be one fish, or my finger if there be two? If one, which one is the real one, which the illusion? Even a camera would show two fish, yet a balance would show but one. And the whole of human experience reveals the same doubtful character of the testimony given by the senses. Unless we were to arbitrarily endow ourselves with an intuitive power of seeing with an internal eye realities given by the senses, we must admit that if only what the senses report is to be accepted as fact then we are poor indeed in realities. Heracleitos must have been right, for the world of the senses is ever shifting, ever new, always a swiftly flying present, and full of contradictions.

Is reality then to be found only in our inferences and deductions from the phenomena of experience? From the observations of his predecessors Ptolemy inferred a structure of the universe which is complete, accurate, and can never be contradicted by nature even if nature should turn out to be full of discontinuities. For his system was built on Fourier series, though he was unaware of the fact, and even discontinuities do not bother Fourier series. Yet we accept as the reality in the structure of the universe the Copernican system. Can then inference give us unreality as reality? Maxwell proved that electricity was an ether stress and discovered wireless waves merely by inference. Yet there is no ether and a wireless wave is a wave with nothing to wave. Indeed it makes no difference whether we suppose there is an ether or

that there is not an ether. Where then is reality in inference? Is the inferred rotation of the earth a reality or merely a convenient fiction? When we see the Brownian fireflies in the ultramicroscope, are the flashes reflected from the surface of a molecule, or from the modern complicated system called an atom. The latter statement is much like saying the solar system reflects light. Is color real, or is the reality only a number attached to a periodic phenomenon which has lost even the tenuous phantasm called an ether as its support? Is a symphony only imagination, while the reality is a series of complicated pulses propagated through the air? Is a cube real? For thousands of years men have studied the cube and always reported the same facts and theorems about it. Inferences may not give us reality any more than sense-perceptions. We find the same contradictions everywhere and not merely confined to physics or chemistry, biology or psychology. For long ago the Sphinx crouched no longer in stone:

> She melted into purple cloud,
>   She silvered in the moon;
> She spired into a yellow flame;
>   She flowered in blossoms red;
> She flowed into a foaming wave;
>   She stood Monadnoc's head.
>
> Thoro a thousand voices
>   Spoke the universal dame:
> "Who telleth one of my meanings,
>   Is master of all I am."

Perhaps here is the way out of our difficulties. If we can unriddle the problem of reality in one instance we may hope to do so in others. Therefore we turn to the universe of the mathematician, for here indeed the problem has been solved, and what reality is, what fiction is, can be ascertained.

II

Mathematics is a vast world of objects and their transformations with both static and dynamic features, intricate and tangled, yet systematic and ordered to a degree far greater than any other known world at present. It is a world created by the mind of man, partly in order to enable him to handle the numerous phenomena he studies, but mostly for the elegant beauty of the objects. The mathematician is an artist who works in a more subtle material than paint or stone, or even tone and words. In recent years he has busied himself with a consideration of just what kind of world he dealt with, and he finds he does not discuss the natural world, though he may receive suggestions from it, but that he creates a world and that the objects of this world grow more numerous day by day, more elaborate, more interconnected, and

yet as time passes this world is permanent and becomes a heritage of the race. The mathematician has concluded that man has always exercised this creative function of his intelligence, indeed perhaps that is the chief function of intelligence. He is convinced that the unknown is merely the uncreated, that uncomprehended complexity is merely chaos and not complexity at all. We can not stop to consider this conclusion of a large number of the masters in this branch of human knowledge, but we may notice just a few of the lines along which this creative evolution has proceeded.

Before the pyramids reared their vertices toward the Egyptian sky, before the valley of the Euphrates was cultivated for its grain, long, long before the first record of history there were men who fought the wild beast and took from him his skin; who fished the lakes and carried home their catch; men who met each other and compared the objects they had gathered. Desire for the other man's booty arose, and when they did not fight for the possession of the treasures, they bartered for them. A skin for a carcass, a small skin for a fish, a handful of fish for glittering stones or perhaps for curiously marked shells; at first a direct handing over of object for object, but later a tally of the objects, a conception of the fact that the parts of the body could represent the collection to be bartered—such was the beginning of the haughty aristocrat of mathematics, the theory of numbers. The Zuñi tallies with his fingers successively and says: the beginning, taken with the first, the middle of the list, all but one, the lot, one with the lot, another with one and the lot; the Bugalai says, little finger on the left, second finger, middle finger, index, thumb, wrist, elbow, shoulder, left breast, right breast; the Australian knows but the single thing and the couple; the Polynesian counts by pairs, by quadruples, and by tens, hundreds, and thousands, for he laid out small objects by pairs, one in each hand, and breadfruit he ties in pona of four each, so that takau means ten, twenty, and even forty; the Maya counted by scores and scores of scores. In this simple manner was suggested to the mind first the correspondence of the tally, then the image of the tally, and finally one brilliant day there flashed into existence the concept which dispensed with the tally and the image. At first the primitive bookkeeper kept his accounts by tallies of some sort, but in the course of time as transactions were more complicated, he was forced to invent a two, a four, a five, a ten, a score, a hundred, a four hundred, even a million, which did not consist of the tallies nor of the parts of the body that were used to tally, but of real abstract numbers. For these he invented arbitrary signs, which no doubt were crude imitations of the objects he had used for tallies. When we reach civilization's story we find the Chaldean astrologer counting by sixties, and writing his numbers by the principle

of position of the symbols, so that he has even sexagesimal fractions, a very elaborate system indeed. Ultimately man was compelled to study this world of number that he had thus created, not as a sort of distillation of many trades or barters, nor as a shadowy composite photograph of the various pairs, quadruples, tens of millions, that he had seen or handled, but as a sharp and definite concept of an object which could not be perceptually visible, but which nevertheless existed in the imagination as a real object. The mathematical imagination, in other words, had begun to create the series of objects which through long eras has become a tremendous world almost commensurate with the world of phenomena. We may easily imagine the primitive bookkeeper keeping tallies on a stick, just as many of the ignorant of our own time do, with big notches for the tens, and very large ones for the hundreds. We may imagine their admiration for the genius who first invented symbols for the tallies and kept the count in his head, taking the objects to be bartered by fives or even tens instead of by ones. We may speculate about the prehistoric period during which these symbols grew in different places into systems of notation, in Chaldean cities becoming the very intricate sexagesimal scale which was useful to the wealthy Babylonian as he counted his millions, and to the astrologer who scanned the jeweled sky for mystic information. The invention of such a system of objects with their multiplication table, addition table, notation of a very effective type, the ability to calculate by means of this invention, and thus to dispense with the handling of large groups of objects, was surely a brilliant inflorescence of the human mind. It eventually dawned upon the mind that in these numbers alone there was a world very well worth the study, and while Cyrus was seizing the treasures of Crœsus, while the Cumean sibyl was burning her prophecies by threes in front of Tarquin, Pythagoras was instructing his initiates in the mysteries of even and odd, and even-odd numbers, perfect numbers, the harmonic mean, squares and rectangular numbers, squares whose sum by twos is a square, and other properties of these purely mathematical objects. They hoped indeed to explain the whole universe by means of these properties alone, just as many another philosopher since has hoped to resolve the universe by means of other mathematical creations.

An unlucky day for them, however, and a lucky day for us came when it was discovered that the diagonal of a square could not be represented by an integer, no matter what number was taken for the side. Here was a downright failure to explain even mathematical objects by numbers, that is the natural numbers, and all hope of extending them to the universe vanished. It was a lucky day, however, because it suggested to the mind that the list of mathematical objects

be enlarged by the creation of the irrational numbers. These include not only the radicals, like $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{2}$, etc., but infinitely many more, such as π, e. Up to the present time no need has been felt to create a further addition to the list. Should the time come, however, as there are some indications that such a time is almost at hand, when it will be necessary, in order to deal with phenomena, to have a set of numbers of higher dispersion, let us say, they will be easy to create and will be readily studied. The modern theory of pointsets and of ensembles in general will take care of this. With the creation of the irrational disappeared counting on an abacus, or tallies of any kind. Concrete objects are useless in handling these numbers, and the mathematician must use his inner eye.

For more than two millenniums the world of number had no evolution. Kingdoms rose and fell, wealth was gathered and vanished to the corners of the earth. Learning of one kind or another made some advance, but the mind was content with its numerical system as it was. Then while Thomas à Becket was quarreling with Henry II., while the crusaders fought about Jerusalem, on the banks of a lotus pond in India Bhaskara watched the swaying lilystalks, and the golden bees that settled in the flowers, and wrote a treatise on "The Beautiful Science, *Lilavati*." Perhaps he was not much more than a compiler, but at any rate we find in his book a new and startling kind of number: numbers that were at first called less than nothing, negative numbers, numbers of which he says the people do not approve. These numbers arose just as did the natural numbers, in the attempt of the human mind to manage the world of its experience. In trying to solve quadratic equations the mind had found it necessary to invent a new kind of number, for sometimes the equation could not be solved without using what we now call negative numbers. Nowhere in all the barters and the mercantile transactions of the past had there been anywhere found a negative number, any more than there is an irrational number to be found in nature. These had arisen as necessary to satisfy the symmetry and harmony of the mind's own world of number, and for purely conceptual reasons. True, in the course of time it was found that the bookkeeper might use these numbers in his credit accounts, if he so chose, but for many centuries they were considered as purely fictitious and of doubtful utility.

The Americas were discovered, and while DeSoto was exploring the Mexican deserts another unscrupulous adventurer was exploring the deserts of algebra, finding and passing by as unprofitable one of the strangest flowers of the field of thought. The square root of minus one, a more daring conception than had yet burst into bloom in all the history of human knowledge, after centuries of slow evolution of the flora

of arithmetic, sent up a solitary stalk and spread its petals to the brilliant sun of the renaissance of thought. Shunned at first as if it were the work of an Aztec sorcerer, for three centuries it slowly spread, until in the modern theory of functions of a complex variable it began to be cultivated for its far-reaching uses. A creation directly from the mind of the race, never discovered in trade, nor in any application, looked upon as an impossibility for centuries, still called imaginary, it has become indispensable even to the practical man. For listen to the wireless call for immediate help, and know that therein is the essence distilled from this once unknown, uncreated flower. The years passed to the number of more than two hundred and fifty, before even a geometrical application was found for this imaginary number and its kind. Certainly we have here at last an indubitable creation of the mind of the mathematician. And, almost 300 years after its appearance, the brilliant Hamilton announced a new variety of number whose inflorescence is clustered in tetrads of imaginaries, and whose beautiful symmetries are the delight of all privileged to perceive them. Since his time species of number have appeared bestarred with imaginaries of very bewildering types, hypernumbers of almost infinite variety, and the mathematician is aware fully at last that he can at will, like Prospero, summon these magic flowers from the chaos of the uncreated, but unlike Prospero's cloud-capped towers they do not vanish with a wave of the wand, rather they spread luxuriantly and bloom superbly, and from them we extract elixirs and perfumes.

We turn back the record again to the age of the Seven Wise Men of Greece, to the beginning of the geometry that was destined to be one of the glories of early Hellenic thought. Whatever it may have been under the great thinkers of Chaldea and of Egypt, it is here that we first find it spreading wide its theorems. Indeed so marvelous was the development of the geometry of Euclid and his successors, that for two thousand years it seemed as if here indeed mathematics had met reality and conquered it. The irresistible power of logic seemed to open all the secrets of space, and it was as rank heresy to question the *a priori* character of geometry as that of theology. Galileo horrified the authorities with his assertion that the earth moved, and a century later a timid Roman priest undertook to show the heresy in supposing that every line had several parallels through a single point. He failed mathematically, his name is almost forgotten, but unwittingly he too had seen the first single flower of another new world. Many decades later Lobachevsky, bolder even than the great Gauss, deliberately set forth in this new world and found that the journey was pleasant and beset with no precipices. Riemann a little later created a third world of geometry, which has in it no parallel lines, no similar figures, nor any solitary wastes of

infinite space. And these three mutually exclusive worlds are now at hand, incompatible with each other, each as infinitely logical as the others, each as fully capable of being the reality of the space we face every day as the others. Riches indeed we now possess, far beyond the dream of Parmenides, for even though the stately architecture of each of these worlds is far more abiding and permanent than the Parthenon itself, yet the three can not exist together, and experience is powerless to tell us either which one is the geometry of the world we live in, or to explain then whence arose the other two. To state the whole matter more accurately, we are forced to conclude that neither one of the three is a reality of the sensible experience we have, but is purely and wholly a creation of the mind, which it may apply to space to some extent, but which is after all neither *a priori* in the mind nor *a posteriori* to fact. They are the result of that quality of mind which refuses to be the sport of the winds of destiny, tossed about in the whirl of phenomena like a dead leaf, and equally refuses to be the prisoner of a granite objectivity, crystallized by the pressure of the eternal ages. The face of the sphinx, whether quarried out of the foundations of the world or moulded out of the flowing cloud, is not fashioned by nature at all, but by man.

There is in geometry, however, a still more striking verification of our thesis. This is the invention of four-dimensional space, and in general of N-dimensional space. Even Aristotle conceived of more dimensions than three, for he concludes that there is no transfer from solid to another kind, similar to transfer from area to solid, and Ptolemy undertook to deduce this result. Up to the period of the Reformation algebraic equations of more than the third degree were frowned upon as having no real meaning, since there was no fourth power or dimension. But about one hundred years ago this chimera became an actual existence, and to-day it is furnishing a new world to physics, in which mechanics may become geometry, time be coordinate with space, and every geometric theorem in this world is a physical theorem in the experimental world we study in the laboratory. Startling indeed it is to the scientist to be told that an artificial dream-world of the mathematician is more real than that he sees with his galvanometers, ultra-microscopes, and spectroscopes. It matters little that he replies, "Your four-dimensional world is only an analytic explanation of my phenomena," for the fact remains a fact, that in the mathematician's four-dimensional space there is a space not derived in any sense of the term as a residue of experience, however powerful a distillation of sensations, or perceptions, be resorted to, for it is not contained at all in the fluid that experience furnishes. It is a product of the creative power of the mathematical mind, and its objects are real

in exactly the same way as the cube, the square, the circle, the sphere, or the straight line. We are enabled to see with the penetrating vision of the mathematical insight, that no less real, and no more real, are these fantastic forms of the world of relativity, than those supposed to be uncreatable or indestructible in the play of the forces of nature. Exotic orchids of the human mind, brilliant with hues no painter ever saw, perfumed with a fragrance that no chemist can extract, beautiful with a symmetry that no draughtsman can depict, fascinating with suggestions of undeveloped powers of the human soul, the creatures of this world have shown man definitively that he is superior to space and time and given him a freedom that is beyond even his highest dreams.

Again, with forceps whose delicacy is infinitely greater than that of the tools of the most skilled workman, the mathematician has put together points like glittering beads, on golden wires whose curve is nowhere viewed with the physiological eye, for these curves may have nowhere a definite direction, they may occupy every point of a square area, they may leave empty positions on the wire infinitely numerous, they may even leave every bead at a finite distance from its neighbor and yet occupy every position, they may have direction, but yet nowhere have a definite curvature—in short they are of such intricate workmanship that it is impossible to deduce their properties by inspection. No physiological experience ever furnished any knowledge of these delicate jewels, no brain-cell or ganglion is their case, yet they are among the finest ornaments of Mathesis, Sophiæ Germana. The creation of lines of this type, or from an analytic point of view, of functions of this type, whose properties are not only not intuitively evident but in many cases are directly contrary to what one might expect intuitively, is another indubitable example of the fact that mathematics creates its objects and studies them with an internal eye whose analyzing power is increasing day by day, beyond all limits of light-waves, or intra-molecular structure, beyond all imagery, with more certainty, however, than even the best physical apparatus can enable the physicist to handle the invisible, intangible, and unperceived ultra-violet or X-ray.

We might proceed to inspect other divisions of mathematics and should find always the same conclusion. In every direction in which we might travel we should always come to the vague, the chaotic, the tangle, the unformed, but under the magic wand of the mathematician we find the vague assuming form, the chaotic appearing in order, the tangle turning into a beautiful lacework, the unformed showing first a line, then a net, then structure. Wherever the mathematician meets nature and follows her suggestions he idealizes the phenomena into something intelligible, he paints a picture of a reality he creates

as having in it a structure which matches the phenomena. Measures are made exact, laws become definite, terms are dazzling with new meaning, formulæ become alive. He comprehends physics in the calculus of variations, this in turn he includes in the functional calculus; he reduces mechanics to geometry and geometry to analysis. He solves problems in phenomenal space by his imaginary space, and calculates real numbers by the square root of minus one. A wizard, nor a sorcerer he, for his results are abiding. He is an artist who molds the phenomenal world into forms of statues and makes even more beautiful figures out of nothing at all. He is like the artist of the Beautiful in one of Hawthorne's inimitable tales, who made a model of a butterfly, so delicate in mechanism, so perfect in poise, as it fluttered around with its purple and gold-speckled wings, that the observers fancied it must be a living butterfly. The beauty faded and the mechanism lost its power on the finger of the skeptical watchmaker, who appreciated only his mechanisms of wheels, it glowed and became vigorous on the finger of the naïve child, it delighted the blacksmith, the builder of engineering structures. When the child clutched it, fancying that the beautiful toy might be his own selfish possession forever, it was crushed into a heap of glittering fragments. The artist placidly looked at the destruction, whence the beauty and the utility had gone forever, undisturbed because he knew that he could at will create even more perfect mechanisms, not because they would be more like living creatures, but because they would be more beautiful. Even though not butterflies, they delighted man more than the butterflies.

III

We turn to the consideration of the theme, what is the real in science? with the meaning of the real in mathematics as a guide to the proper answer to the question. We may begin with the oldest part of physical science, going back twenty-two centuries, to the city of Syracuse, where the conquering hosts of Rome have sacked the city, and we see an enraged Roman soldier attack and kill a dignified old man who has been busy with diagrams and surrounded by mechanical inventions of various kinds—Archimedes, in fact, the founder of mechanics. Him we remember indeed while the names of the Roman conquerors have sunk into a just oblivion. His determination of the gold and silver in the crown of King Hieron is mentioned in many a school reader, and the joyful cry "Eureka" has echoed again and again in the laboratories of the world. He idealized the lever and worked out the properties of this imaginary weightless, rigid, segment of a straight line. Since that first bold substitution of an ideal, axiomatic lever for the bar of wood or iron, mechanics has again and again idealized the objects it meets

in nature and to such an extent that rational mechanics is really a branch of mathematics, while applied mechanics points out how closely approximate the results are when attached to material objects.

A stretch of seventeen centuries follows with little advance, up to Leonardo da Vinci, artist and scientist, who with Stevin of Bruges, begins the development of statics. They were followed very soon by Galileo, the first to conduct extensive experiments for the purpóse of bringing his idealized phenomena into close harmony with the material phenomena. We see him dropping shot from the tower of Pisa, rolling balls down inclined planes, counting the oscillations of the chandelier in the cathedral, finally with a flash of intuition announcing that a body in motion will remain in motion, and at rest will remain at rest, unless acted upon by some force, thus creating at one stroke two of the entities of modern physics, inertia and force. Bold indeed from a few experiments, on a few objects, to announce for all the world such laws, and to assert that all kinds of matter possess something called inertia! Inertia and mass were long synonymous, and the scientists soon accepted mass as the quantity of matter itself, whatever its kind. But we have come to see a difference between mass and inertia and we may now ask if an electron has inertia, and what kind of thing inertia must be that it is possessed in common by gold, radium, helium, hydrogen, uranium. Does the nucleus of an atom have inertia, and is the nucleus nothing but inertia? If one were on the moon, or the sun, or the lost Pleiad, would these same things possess inertia? Who knows if a particle moved in a line for billions of years in purely empty space whether would it come to rest, or always move on and on without limit? If the inertia of an electron is only apparent and due to the fact that a moving charge generates a field which retards the charge in its motion, why may not all inertia be of the same character, and thus be as fictitious as the added mass of a sphere moving in water?

The year of Galileo's death was the year of Newton's birth, titanic successor to a giant predecessor. His intuition was even bolder than Galileo's, for he conceived the entire universe as knit together with ideal threads. This ideal object he called gravity, the most universally present force, and the one we are to-day most ignorant about. This mysterious object of dynamics reaches to the uttermost confines of the universe so that the quiver of an eyelash is followed by a shivering of all the particles in the magnificent sweep of creation, yet so tenuous is it that nothing seems to interfere with its passage. The huge ball that carries humanity as well as the crater-rimmed satellite that sends its soft glow down to summer-strewn banks, cast no shadows in space under the stream of this penetrating essence of Newton's mind. The radiation of the brilliant star of day is stopped completely by either of

these, and the long shadow trails for millions of miles like a gigantic crayon which marks every object it touches; the X-ray finds its path even among the molecules of the crystal, yet is finally stopped by all kinds of substance; but this creation of the great genius of mechanics—what shall we say of it? The electron in its dizzy spinning and the stately Neptune in his many-yeared cycle, the sun in his wanderings and the nebula in splendid isolation, all alike are mastered by this immaterial, spirit-like, purely ideal creature. And is gravity a kind of matter? So Le Sage thought, but few would agree with him. Does gravity pull on the electron? Who knows? Newton did one thing whose import was not at first perceived, namely, to lay down a universal law that mass, or inertia we may properly say, is measured exactly by acceleration, and in fact accelerations are the only entities that need enter the equations, inertia becoming a mere numerical ratio. Is it possible then that the rock-ribbed hills, the stately palaces, and the gauze of the comet, all alike, are but such stuff as the mathematician dreams of, and the reality is a ratio? Indeed the modern physicist thinks he has shown that even this ratio is variable, depending upon the velocity of the moving point which has been substituted for the material particle. Gravity is not matter, and inertia is a ratio! But this virtually is saying that the world of mechanics is purely an ideal world, created by the physicist's mind in precisely the same way that Lobachevsky or Riemann created a non-euclidean world. Neither is more real than a cube, and either is real in the same way a cube is real.

Since Newton's day the other conception, force, has been extended to a great variety of phenomena, so that we have had force of cohesion, force of electricity, force of magnetism, force of chemical affinity, force of elasticity, and still others imaginable. Mysterious fingers were at every point in space ready to clutch the appropriate object of their desire, were it particle of matter, electron, magneton, molecule, or atom. The force of gravity was pulling all the time on every one of us to keep us from falling off the earth, the cohesive forces were keeping us from being shattered into dust before our time, the chemical affinities were at work building up the compounds of which we consist, and the tremendous play of these giants struggling against each other throughout the universe preserved us from chaos. They represented a premodern transformation of the nature gods of early history. For the modern physicist has abolished the once real forces, and no longer imagines space as full of the invisible fingers the giants once had. Like the childish dreams of fairies who did things secretly that man could not do, these naïve conceptions of the physicist and chemist have given place to others which are supposed in their turn to represent reality. Is there any better example needed of the creation of an ideal

reality which was relegated to the museum when no longer useful in the handling of phenomena? Inertia a ratio and force expressed completely in terms of change of position in space—the dream of Pythagoras realized in one, that of Heracleitos in the other. Where is the unchanging reality Parmenides asked for? Is it true that we can only say of two dust particles that they gravitate, that is move towards each other, of two electrons that they move away from each other, of atoms in a benzene molecule that they form a stable group, and, though stable, are in an incessant and intricate waltz figure forever shifting its diagram? When a huge charge of nitroglycerine explodes and shatters the surrounding objects, can we say nothing more than that the atoms of nitroglycerine separate under certain conditions in the neighborhood? Is it possible to reduce all physics and chemistry to the calculation of a function of each point in space called the potential of that point, a function depending for its value upon all the points in space where matter or electricity or other object is situated, a function to be determined in all its non-singular points by the position and certain numerical values associated with the position of all its singular points? We have used the gravitation potential, the electric potential, and the magnetic potential, and we might add the chemical potential, as a matter of convenient mathematics, but what if potential is the only reality? At least it is as real as any other conception we have of nature, yet no one has ever thought of it as other than a mathematical term.

The year Illinois was admitted to the galaxy in the blue sky of the American flag, was born the man who should not only revolutionize many of the conceptions of physics and chemistry, but should revolutionize the whole of natural science. Before his time there had existed an imponderable, invisible fluid called caloric, to which was due the phenomena of heat. We still have treatises on the flow of heat, in which the equations refer to a continuous fluid. But the famous experiments of Joule brought forth a new reality called energy and the law of the conservation of energy, said to be the grandest achievement of the human mind. Sublime substitution of the more unknowable for the unknowable! Sublime determination of the one great reality of nature which persists through all the swift transformations of atom, molecule, cell, or universe, from the meanest flower that blows to the most infinite nebula seen or unseen! Energy is that which can neither be created nor destroyed, the great god Proteus, changing in a flash his form so that he may not be recognizable, who may hide utterly so that his presence is unsuspected, yet in a twinkling may let loose his awful power and rend the visible universe. Energy was nascent in the phlogiston of the eighteenth century, for phlogiston was what escaped during the burning of substances, but phlogiston was a substance, and when

it was proved that it had no inertia, it was also stored away with the other models of man's creation as no longer of use. Energy can not be measured, neither weighed, nor seen under the microscope. Its presence can not be detected by an apparatus. Like the velocity of the earth, we know nothing about it directly. It is not involved in any of the phenomena of sense. It is not an inference from the observations nor experiments of the laboratories, such as the velocity of the earth may be. It is a direct creation of the human mind as much as the square root of minus one. When we have ascribed certain physical or chemical changes to the transformation of energy, the residue which is unaccounted for is ascribed to some new form of energy. We invent these forms as we need to make the law of conservation of energy hold good, and we demolish them as we find that we can account for phenomena with fewer forms that energy may take. We observe motion and we ascribe energy to the moving points, kinetic energy, depending upon the square of the velocity. The point may stop and the energy be gone, but then we say it has become potential. We always have an unlimited supply of kinds of potential energy to draw upon. We have potential energy of gravity, the energy a stone on a cliff has because it is on the cliff; the potential energy of an electron, the energy it has because of its situation as regards other electrons; the energy of an electric field, or a magnetic field, both purely immaterial mathematical fictions, is potential; the energy let loose by the high explosive shell that screams over the battlefield was potential energy. But most astonishing of all we may describe mass as merely energy per velocity per velocity. The most unknown reality of nature enables us to define that which we might suppose to be the most known reality of nature. Yet it is not matter that interests the future of the race, but energy. By it we live and move and have our being, and when we can no longer control energy we must perish. Profligate sons of Pan, we gaily spend the stores of energy slowly accumulated by mother earth, nay more we almost deliberately waste the stores of obtainable energy. We see already the darkening horizon of the future when coal will be gone, oil and gas fields exhausted, and we are even now desirous of robbing Niagara of its beauty that the sale of energy may fill pockets with gold. We see many a Swiss valley with a glacier at one end and nitric acid at the other. We build dams like cliffs to utilize the kinetic energy of the white coal, we rob the winds of their store, and we would chain the ocean wave to a treadmill.

We may even go further and say that there is no potential energy, but that all energy is the energy due to motion, even though the moving points may be hidden so that their motion is simply assumed. We have done this in explaining where all the heat energy goes when a gas

receives heat. Some of it causes the molecules to move faster, but some of the heat, and in complex cases much of the heat, becomes energy of rotation of the molecule and energy of the motion of the atoms within the molecule. In fact, if enough heat is supplied the molecules may be broken up. When we come to the atom, however, which is itself conceived to be a system of swiftly rotating electrons with a positive nucleus, we find that heat can not be transferred to it, in fact so far man has been unable to interfere with the structure of the atom. Yet in the case of radium every one knows that we have as unstable a system as a nitroglycerine molecule, and that the atom breaks up by expelling every so often electrons, and every so often helium atoms. The amount of energy let loose in this way is enormous, in fact the whole earth with its volcanoes and geysers, with its earthquakes due to internal explosions, is getting hotter and hotter from the radium explosions within, and the accumulation of energy may sometime cause the entire planet to explode. What kind of a motion is it that can be so vigorous as this, inside the infinitesimal space that an atom occupies? If the potential energy is after all only the energy of these invisible motions, how enormously fast it must be!

In connection with the transfer of energy, as from the sun to the earth, we find the creation of a medium called ether. It seemed inconceivable that energy, if it be only motion, could get from the sun to the earth without anything to move in the interval. At a speed of 185,000 miles per second this medium transmits radiation of all kinds, whether light, heat, or the wireless wave. The auroras of the northern sky due to the magnetic energy that arrives from the sun when some cyclone of terrific power shoots out its winds of intensely hot hydrogen for hundreds of thousands of miles, the ceaseless flow of waves of energy as the electron circles around its nucleus billions of times in a second, waves that impinge upon matter like the waves of the ocean upon a mass of rock, capable of setting matter in motion by their pressure, all these surely need a medium to transmit the energy if it be motion, that is, kinetic. But the modernmost science sees no further need of this medium and it may also be put away with other discarded models the mind has made, for if it does not exist save as a conception, how can it be a reality of any type but mathematical? In fact it is still simpler to consider that energy itself moves in empty space, and that a wave consists of a current of energy that waxes and wanes, rising to a maximum and sinking to zero several billion times within a second. This current of energy has a momentum, which is the quotient of the energy density by the velocity, and a mass, which is the quotient of the momentum by the velocity. Thus an electron is reduced to moving energy, this energy having a central point which it does not

reach, the intensity of the energy varying as the inverse fourth power of the distance from this center, the center being surrounded by a sphere of discontinuity at which the energy becomes zero on one side and a very great value on the other. If it be possible then to reduce all electrons to energy distributed in space in this manner, and all matter to aggregates of electrons, we obviously have come finally to Parmenides's vision of the one substance of being. And we might also say that we have realized Heracleitos's vision of the Fire whose fixed measures are always being kindled and fixed measures going out. What a perfect chaos of change the universe becomes, for at every point we see the streaming energy from an infinitude of centers, flashing through space with the enormous velocity of light and producing at each fixed point such an intricate function of the time that we fail utterly to get any picture of the phenomena! And the rotating electron, so rich in this cosmic matter, which it radiates at every whirl with tremendous prodigality, must inevitably slow down as it loses its energy, and ultimately become stationary, and thus nothing at all, unless from some other blazing center it receive a new life. Besides this there are all the moving clusters of energy traveling through space on their own account, with no such attached discontinuities as electrons. Whether energy is granular or continuous is of little import as far as our problem is concerned, the reality of energy being the only question, and if this reality reduces to a non-substantial energy as the only being then we have a reality whose difference from the mathematical type is not obvious.

We have frequently mentioned the electron and the nucleus, other creations of the human imagination, things we can not ever hope to see, existences which we postulate in order to account for phenomena. The new science of radioactivity has produced these objects. The electrons are of the nature of negative electricity, the nuclei are positive, so that the substance electricity has been put in the museum of antiquated models of thought, and neither the one-fluid idea nor the two-fluid idea of electricity is anything more than a useless mathematical model. When will the day of doom of the electron and the nucleus come, and these too go the way of all such constructions of the intellect? And yet when we see the scintillations of the fluorescent screen near a particle of radium it is very convenient indeed to imagine the terrific bombardment of the high-power guns of the minute atoms as they explode their smokeless powder. A nucleus may be a system, but whether an electron is a single thing or a system we can not imagine, and for the present the model is that of a sphere or an ellipsoid, though upon occasion who doubts that we would cheerfully make it over into a whole solar system? We are just now endeavoring to discover whether we

had better endow the electron with inertia or not, with weight or not? The nucleus seems to carry all the weight of the atom.

The chemist has also had his turn at playing with the toys of mind. His molecules he long ago considered to be a handful of round pearls of thought, which he strung into necklaces for the adornment of Scientia, necklaces which consist of chains, with pendants, fantastic clusters, and other figures so mathematical that the great Cayley studied the whole subject as a piece of mathematics. His pearls have turned out not to be pearls, but to be tiny solar systems, so that Scientia is now adorned with strings of universes. These tiny solar systems contain valence electrons whose attractions and repulsions have replaced the chemical affinities. We need not elaborate on the situation, but we must wonder what the diagram will look like a thousand years from now. He has dispensed with eighty or more kinds of matter, save as a matter of convenient language, and is now describing the eighty or more kinds in terms of a few nuclei and electrons. Perhaps he can dispense with the nuclei if he can imagine an intricate enough arrangement of points to take its place, and let the whole universe be described in terms of energy alone, or of mere granules of ether alone, with high velocities to furnish the phenomena of change. The atomization of mathematics has passed the meridian, but that of science is at its zenith.

When we turn to the phenomena of life, the study of the biologist, we find the realities no different. At present he is endeavoring to his utmost to express all these in the same terms that the physicist or the chemist uses, partly in the vain hope that when they are so expressed he can predict what kind of an animal, whether an elephant, a man, or a bacterium, will be the result of bringing together so many molecules of this or that, and so much energy of this or that type. He sees perhaps that to create magnetism where none existed, all that is necessary is to start an electron or even a mere mathematical line of electric force moving, and presto, the magnetism is at hand; so he hopes to rival this magician's trick with one of his own, and by setting electrons and atoms into some kind of motion, arrive at life. Perhaps life consists of such a combination! It may be that there was first a universe filled with granules of nothing at all, mere points, that moved with the velocity of light in all directions, thus having tremendous kinetic energy, or just energy let us say. Perhaps some of these lost some of their velocity, not for any reason or on account of any cause (for such terms are philosophical and forbidden in science), but they just did. The energy being indestructible, exercised its protean prerogative, and changed its form, becoming potential energy, and the thing that now moved was an electron, moving potential energy. Perhaps these electrons in turn congregated accidentally into systems, some of which turned out to be unstable and broke up, some of which were stable and became what we

call atoms. Perhaps the atoms in their Odyssey found other atoms, and with no cause whatever, they joined themselves together because the combinations were stable, that is, did not break up. Perhaps these molecules, as the æons rolled along, also found certain groupings were not broken up, but persisted, always with the accompanying accumulation of potential energy, the potentiality being merely the fact that they did not break up. Perhaps it was the fate of these multimolecules of the colloidal state in the strange caprices of chance to find that some of their combinations were willynilly stable, and were later to be called cells by the scientists. Perhaps the cells became organisms and living creatures. The living creatures during untold and unimaginable periods of time have themselves become more and more intricate, and some of them now group themselves into societies and nations. Yet somehow in all these wild saturnalia of chance, it seems the steady drift has been to form the more and more complex, the more and more highly organized in which the adjustment of the electrons, the atoms, the molecules, the multimolecules, and the cells, is more and more delicate, but, strange to say, more and more persistent. Yet the biologist must admit, what he is reluctant to admit, that when we get to the complicated process of cell-division we must label some of the energy biotic energy, for it is unlike the previous forms. This ought not to be surprising, for if the mere motion of electric energy in space somehow creates instantaneously a certain amount of magnetic energy, neither being quite statable in terms of the other as to kind, we ought not to be startled if in all this long chain of evolution, there should come some day a further form of energy. Convertible of course into the other forms of energy, otherwise it would not be energy, but yet different in kind. We can not help wondering too what peculiar numerical combination always loaded the dice in the game, so that the *complex* was the inevitable result. In mathematical probability we expect homogeneity to result from the chaotic mixture of any set of points with any kind of velocities, not heterogeneity; and even the second postulate of thermodynamics says that entropy tends to a maximum, which is simply the same as saying that in a big enough chaos, everything tends to homogeneity, mediocrity, a dead level. We feel somehow compelled to think of Maxwell's demon as opening the gates for those motions which tended to complexity, and shutting it to those which tended to entropy. So here is a further element that we must put into our model, the feature of evolution, one that so far the physicist and chemist has no need of. This evolution we must call a creative evolution, because we pass from the less complicated to the more complicated. This creative evolution becomes as much a reality as any of the other things we have mentioned. Whatever we are obliged to use in the construction of our model of the universe is a reality of science.

When we no longer need it, it does not cease to have reality, any more than a cube goes out of existence when we study spheres.

The new feature of the model has been called vitalism, a vague term too much associated with phantasms; and entelechy, a more technical term. Entelechy is not more Hellenic than entropy, and hence we need not be frightened at it. It is the formative character of the living thing, that which tends to produce the complexity. The entelechy of a circle is the line which makes the points of the circle into a synthetic whole and not a mere aggregation of points. Some mathematicians, it is true, consider that a circle is nothing but a series of points; some physicists consider that the aggregation of electrons and nuclei is an atom; some biologists consider that the aggregation of multimolecules is a cell; but the modern theory of functions of lines thinks that a circle is more than its points; and the explosions of the radium atom show that there is a stability which may let go and the mere electrons and nuclei cease to be radium; the organizing power of cells, their reproduction, their self-repair, their storage of energy of various forms, their ability to increase their potential characters, whether of energy, motion, or structure, demand that we put entelechy into our image of the universe. What is entelechy? is no more foolish a question, and no less foolish a question, than the corresponding one, what is energy? When we have a way to measure it, it will be used just as much as energy. And it seems obvious that entelechy will have to start with the electron and the atom.

There is still another series of phenomena which the scientist is obliged to study, and in which he might hope to have more success than in those previously mentioned, for they seem to be so immediately close to him that they are part of him. The phenomena of mind ought to be those indeed which the intellect could lay hold upon and strip to their ultimate reality. But here too we see the same procession of models rather than ultimate realities. We must content ourselves with an imitation, or shall we say that what we can call real is only that which we create, and that the real would not be at all if we did not create it? There once used to be in this science a soul, a subtle fluid or spirit, that permeated the whole man, which persisted even after cells had broken down into molecules, after entelechy had ceased to form. We need not pause to study the progress of philosophy and science which annihilated the soul, or rather put the soul into the museum. Mind took the place of soul, a substitution of a more vague term for the vague term soul. Mind was complex, but it was for a time an aggregate of faculties, perception, conception, imagination, judgment, reason, will, memory; and educational systems to-day preserve these divisions. Mind was also constructed out of ideas, linked together in associations which were indissoluble, a vast Pandora box, which occa-

sionally opened and allowed some of its creatures to escape for a time into the air of consciousness, in groups that were too often made up of as many undesirable elements as desirable ones, but always being stored away again in this ever-accumulating aggregate of experience. Mind was later merely an aspect of the central nervous system, and the model became that of the net of nerve cells and their neurons and dendrons— a very mechanical model, and one most facile in "explaining" the phenomena of psychology. If the system split into two sets of cells, isolated from each other, then there were two minds, two personalities in the same body. This model too went its way, to the museum, and the new model was a stream, the everlasting flow of Heracleitos. Like the wraiths of morning mist on the mountain lake, fading away from sight even as they come into view, the wisps of the processes of consciousness are vanishing even as they appear. Just as the biologist might consider that life is merely a series of secretings, circulatings, respirings, digestings, this model of the psychologist's making is only a sequence of happenings. The dream is only the dreaming, the symphony only the playing, the ache only the aching. Like Prospero's magic everything disappears with the thinking. In conclusion we have one more model of mind, which preserves its stability, its entelechic organization of experience into personality, its directive and selective character, its purposiveness, and adaptation of means to an end. Mind becomes a creative agency, and its evolution from the void into the determined, undaunted creature of to-day becomes the story of the universe. Tense with activity, from its bonds it creates spaces and times, from the universe worlds of intellect, from star-dust Pleiades; it makes the winds of eternity carry its wings, it floats on the waves of the stream of phenomena; it touches matter and energy, life and mind, and the Queen of Beauty steps forth, never to sleep again; it plays on the pipes of Pan, and circling electron, blazing Sirius, throbbing cell, and wandering creature, burst forth into music; to intellect it adds intuition, to understanding sympathy, to contemplation creation. No longer does the tantalizing search for the reality of the swiftly fading vision it once had by its flickering waxlight continue, for Psyche has become a goddess, and beholds Eros forevermore.